Notes on the Six Sigma Concept

by
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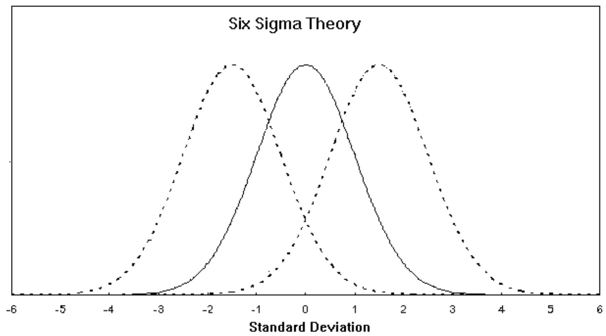

Figure 1. Shows the six sigma concept in a pictorial fashion. This figure shows the normal curve, representing a process (bold line) centered on zero sigma. Motorola's concept assumes that a process can shift 1.5 standard deviations as a regular matter. If the process shifts that much, they argue, the tails of the process would lap over the tolerance limits if the process width were kept at ±3σ equaling the tolerance of the process. To avoid that, they set a target for a process where the limits are narrow enough so that a 1.5σ shift will not shift the edge of the process beyond the tolerance limit. To do this, they equate the tolerance of the process to ±6σ. The tail of the process distribution is not supposed to be closer than 1.5σ to the edge of the specification or tolerance limit. That means that the edge of the process distribution should end at ±4.5σ from the center. The value of 4.5 comes from the value of 6 less the 1.5σ shift.

To compute the area outside x-sigma we use the Normal integral, $Q(x) = \frac{1}{\sqrt{2p}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt$ 0.

This can be evaluated as follows[1]:

Define $R = f(x)(b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5) + e(x)$

Where $| e(x) | < 7.5$ ´10^{-8}

$t = \frac{1}{1 + r|x|}$ 0, r=0.2316419

$f(x) = \frac{1}{\sqrt{2p}} e^{-\frac{x^2}{2}}$ 0

$b_1 = 0.319381530$ $b_2 = -0.356563782$
$b_3 = 1.781477937$ $b_4 = -1.821255978$
$b_5 = 1.330274429$

Then $Q(x) = \begin{cases} R & \text{if } x \text{ ³ } 0 \\ 1 - R & \text{if } x < 0 \end{cases}$

The Table on the next page shows the result of the calculation for selected values of t sigma.

Table of Selected Values of Area Under Normal Curve

Shift Value	% Within Curve	Parts per million
-6.0	99.9999999%	0.0
-5.5	99.9999981%	0.0
-5.0	99.9999713%	0.3
-4.5	99.9996599%	3.4
-4.0	99.9968314%	31.7
-3.5	99.9767327%	232.7
-3.0	99.8650033%	1350.0
-2.5	99.3790320%	6209.7
-2.0	97.7249938%	22750.1
-1.5	93.3192771%	66807.2
-1.0	84.1344740%	158655.3
-0.5	69.1462468%	308537.5
0.0	50.0000001%	500000.0
0.5	69.1462468%	308537.5
1.0	84.1344740%	158655.3
1.5	93.3192771%	66807.2
2.0	97.7249938%	22750.1
2.5	99.3790320%	6209.7
3.0	99.8650033%	1350.0
3.5	99.9767327%	232.7
4.0	99.9968314%	31.7
4.5	99.9996599%	3.4
5.0	99.9999713%	0.3
5.5	99.9999981%	0.0
6.0	99.9999999%	0.0

The assumption of a process shift lead Motorola to specify tolerance limits such that they are ± 6 times the process standard deviation. Their assumption also requires that the process distribution be normal. What they overlook is that the actual process distribution is unknown. Dr. Walter Shewhart, the inventor of the control chart, never required that the process distribution be known, or that it be normal. He used the *model* of a normal curve to establish the operational definition of a special cause. Shewhart used the word “assignable” instead of the more modern word, “special.”

If, in fact, a process shifts at all, we do not consider the process to be in control, meaning that it is not predictable. An illustration of such a special cause situation can be found on page 12 of the Automotive Industry Action Group’s, *Fundamental Statistical Process Control: Reference Manual.* To assume normality of the process and to assume that the process shifts is to admit that the process is unstable. The normal distribution, should it exist has variability built into the system.

When a process is unstable, it is not predictable. Work should proceed immediately to correct this situation. Changing the tolerance limits is not a way to correct an unstable process.

Another factor to consider is the cost involved. Using the Loss Function concept developed by Professor Taguchi, the loss to society (the company and to others) is a function of the mean and standard deviation. In Figure 2 below, both distributions fit the six sigma criteria. The difference in cost is enormous.

For a two-sided loss function, the formula is $L = C \{ \sigma^2 + [x - \mu]^2 \}$

Where C is a constant determined by the shape of the curve

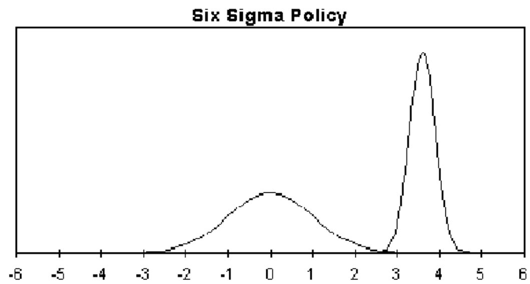

Figure 2. Comparison of two curves fulfilling
the Six Sigma Criteria

We can compare the relative cost of the two curves from the following data:

	Case I	Case II
Mean (μ)	0	3.6
Standard Deviation (σ)	1	0.3
Loss Function = C { $\sigma^2 + [x - \mu]^2$ }	1C	13.05C

Dividing the loss function from case II by case I one can see that the loss is 13 times greater for case II even though both situations meet the six sigma criteria.

The six sigma criteria concept is built on assumptions that often do not exist and are not realistic. Processes that shift any amount are not stable and certainly do not show any evidence of normality. These processes are not predictable. Relying on the six sigma criteria, management is lulled into the idea that something is being done about quality, whereas any resulting quality is accidental. The six sigma policy has a great potential to be very costly, more than is necessary. A far better policy is that of continual, never ending improvement of the process. The continual improvement works on the stabilization of the process and reduction of cost by coming ever closer to the target value and reducing variation.

[1] Adopted from Abramowitz, M. and Stegun, I. A. *Handbook of Mathematical Functions*, National Bureau of Standards 1964 (seventh printing 1968) Par. 26.2.17, page 932